Full-Wave Analysis of Aperture Coupled Microstrip Lines

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Abstract - Two methods are presented for the analysis of aperture coupled microstrip lines. Assuming a quasi-TEM travelling wave incident on the feeding line, an expression for the wave on the coupled line is derived. First the moment method is used and the current on the coupled line is represented by a travelling wave propagating away from the slot. In the second method, the reciprocity theorem is applied to the coupled line. An equivalent circuit is derived and the S parameters are computed. Theoretical results are verified with measurements.

I. Introduction

This paper presents a full-wave analysis and an equivalent circuit for the problem of two parallel microstrip lines coupled by a small rectangular slot in the common ground plane [1]. This circuit can be represented as a four port network as shown in Figure 1a. An input signal applied to port one is partially coupled to the top microstrip line, and this coupled power divides equally between ports three and four, with a 180 degree phase shift. By terminating one or two of the microstrip lines with stubs, the four port network can be reduced to a three port, or a two port, coupler (Figures 1b, 1c).

To date, there has been little analysis on this problem, although related problems were treated in [2], [3], and [4].

II. Analysis

In this paper, the four-port geometry in Figure 2 is analyzed using two methods. The microstrip lines are assumed to be infinitely long and propagating a quasi-TEM wave:

$$H = h_y \hat{y} \tag{1}$$

where h_y is the transverse modal magnetic field of the line. Applying the reciprocity theorem to the feeding line leads to the following relations for the modal reflection (R) and transmission (T) coefficients on the feeding line [5]:

$$S_{11} = R = -\frac{Vo}{2}\Delta v_f \tag{2}$$

$$S_{21} = T = 1 - R \tag{3}$$

where Δv_f is defined as the voltage discontinuity on the feeding line as in [5].

In what we refer to as the moment method solution, the unknown current on the coupled line is expanded in terms of a travelling wave mode:

$$J_x^c(x,y) = I_0\, f^{trav}(x)\, f^{unif}(y) \tag{4}$$

where

$$f_{trav}(x) = \begin{cases} e^{-j\beta x} & x>0 \\ e^{+j\beta x} & x<0 \end{cases} \tag{5}$$

$$f_{unif}(y)=\begin{cases}\frac{1}{w_c} & |y|<w_c/2 \\ 0 & |y|>w_c/2\end{cases} \quad (6)$$

This mode is based on the line eigenfield, and is also dictated by the continuity of H_y at x=0 and the discontinuity of E_x at x=0, and accounts for the travelling waves propagating on the coupled line away from the slot. In the region of the slot discontinuity, however, we might expect a need for some decaying current modes to represent the reactive energy near the slot. As for any discontinuity, one would expect the excitation of additional currents besides the dominant mode current, especially when the two transmission lines have different characteristics impedances. The requirement for such non-dominant current modes was tested by using subsectional PWS modes which resulted in a change of less than 10% in the equivalent admittance.

The electric field in the slot is assumed to be of a piecewise sinusoidal form with an unknown amplitude. As formulated here, the problem consists in solving for four unknowns: R, T, V_0, and I_0. At this point, the additional two equations needed, are obtained applying the boundary condition for H_y in the slot and enforcing $E_{tan}= 0$ on the feeding line. The solution of these four equation system is obtained in a similar manner as done in [5]:

$$V_0=\frac{-2\,\Delta v_f}{2\,Y_{tot}+\Delta v_f^2} \quad (7)$$

$$R=\frac{\Delta v_f^2}{2\,Y_{tot}+\Delta v_f^2} \quad (8)$$

where Y_{tot} is a series admittance to the microstrip line, representing the slot discontinuity. Since the input current was assumed to be 1 amp, the power delivered to each output port on the coupled line is

$$P_3^-=|I_3|^2 Z_{oc}=|I_0|^2 Z_{oc} \quad (9)$$

and the power coupling coefficient C_p is

$$C_p=\frac{P_3^-}{P_1^+}=|I_0|^2 Z_{oc} \quad (10)$$

where Z_{oc} is the characteristics impedance of the coupled line. It appears that the use of one mode in the moment method could lead to some error, if in reality some other modes were excited. For this particular geometry, however this does not happen. Intuitively, the slot field, being parallel to the transmission line axis, creates a reflected wave which is the travelling mode used. This is the case even for lines of different widths provided the ratio w_c/w_f is smaller than the ratio L/W. If the slot were oriented obliquely with respect to the couple line, or the lines were not parallel, additional modes would probably be needed in order to obtain reasonable results.

Based on the relations obtained, an equivalent circuit was derived (Figure 3). The coupling effect is represented by an ideal transformer. For example, the case in which the two lines are identical would correspond to a turns ratio equal to one.

III. Conclusion

Two methods were presented for the analysis of two parallel aperture coupled microstrip lines. The moment method as well as the reciprocity method make use of the exact Green functions,and produced results that are in very close agreement (Figure 4). However, the reciprocity method is computationally more efficient: the CPU time required is about 60% less than that needed by the moment method. On a Cyber 850 computer, the solution based on the moment method required 117 seconds, while the solution based on the reciprocity method requiered only 47 seconds. A four port coupler was built and tested to verify the theory. This microstrip to microstrip transition presents a very attractive property of a thoretical unlimited bandwidth for the S_{43} coefficient. An equivalent circuit which should aid in the design of practical coupled line circuits was developed.

References

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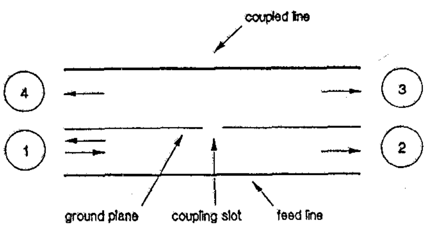
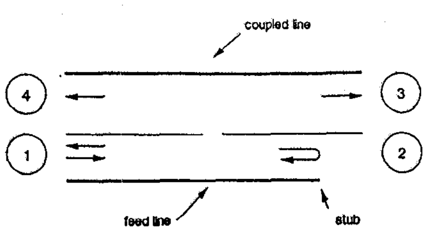
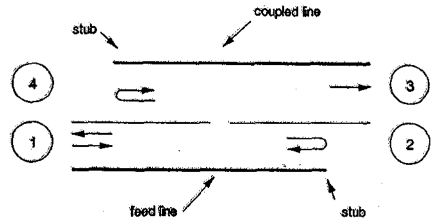


a. Four port-circuit



b. Three-port circuit



c. Two port-circuit

Fig.1 - Aperture coupled microstrip lines

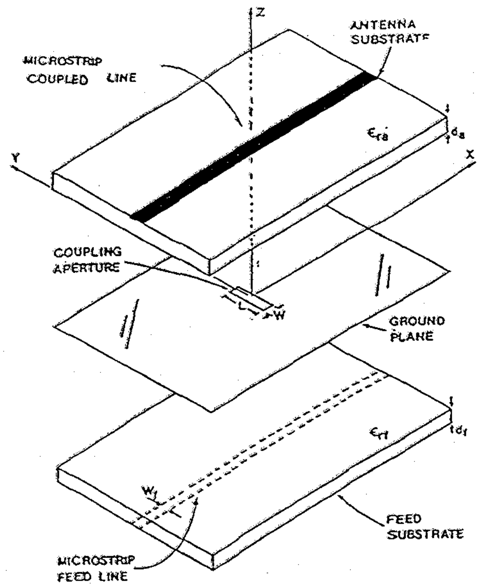
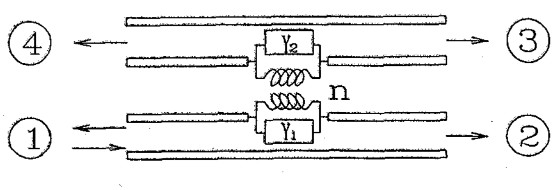
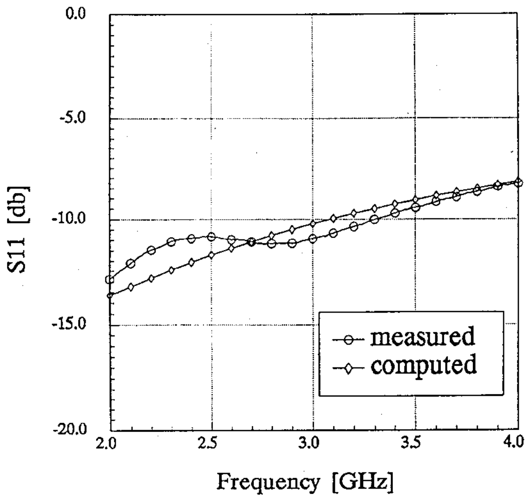


Fig.2 - The geometry of two aperture coupled microstrip lines.

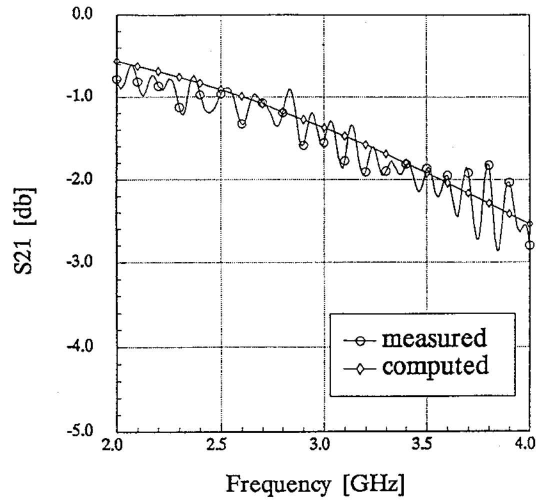


$$Y_1 = \frac{1}{Z_{0f}}\frac{Y_{af}}{\Delta v_f^2} \quad Y_2 = \frac{1}{Z_{0c}}\frac{Y_{ac}}{\Delta v_c^2} \quad n = \frac{\Delta v_c}{\Delta v_f}$$

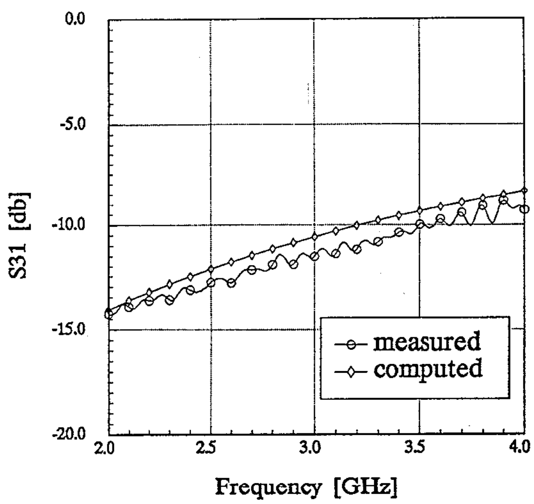
Fig.3 - The equivalent circuit of two aperture coupled microstrip lines.



a. S_{11}



b. S_{21}



c. S_{31}.

Fig.4 - Measured and calculated S parameters (db) for two aperture coupled microstrip lines.
$w_c = w_f = 0.254$ cm, $\varepsilon_c = \varepsilon_f = 2.22$,
$d_c = d_f = 0.0762$ cm, L=1.5cm,
W=0.11 cm